

XX. *Of the effects of the density of air on the rates of chronometers.* By GEORGE HARVEY, F. R. S. E. &c. Communicated by DAVIES GILBERT, Esq. V. P. R. S.

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AMONG the different sources of error to which chronometers are commonly considered to be liable, the effects of the variable density of the medium in which the balance performs its vibrations has, in some degree, been overlooked. That changes in the density of the medium, produce however a sensible influence on the rate of a delicate time-keeper will, I hope, clearly and satisfactorily appear, from the detail of the experiments, now respectfully submitted to the Royal Society.

The investigation of the subject has been undertaken in the four following points of view :

*First*, by subjecting different chronometers to a *less* pressure than that afforded by the ordinary state of the atmosphere at the level of the ocean.

*Secondly*, by submitting them to a *greater* pressure than that afforded by the atmosphere under the same conditions.

*Thirdly*, by removing chronometers from condensed into rarified air, and *vice versa*.

And *fourthly*, to determine how far the rates of chronometers are affected by the ordinary aberrations of atmospheric pressure at the level of the sea.

1. To estimate the effects produced by the first of these

conditions, the chronometers were placed beneath the capacious receiver of a large double barreled air pump, the pressure being indicated by an excellent mercurial guage.

To prevent any irregular effects from the unequal action of terrestrial magnetism, the position of each chronometer, with respect to the meridian, was preserved *constant* during the whole course of experiments.\*

The first chronometer selected was an eight day one, of the box kind ; and which, for the purpose of farther reference, I shall distinguish by the letter A. Its rate for ten days previous to the experiments was steady and uniform, amounting to  $-3''.1$ , the mean pressure of the atmosphere being 30.1 inches ; but when placed beneath the receiver of the air pump, under a constant pressure of 20 inches of the mercurial column, the mean of four days observation gave an equally steady rate of  $-1''.3$ , the chronometer having gained  $1''.8$ , by diminishing the density of the air in the ratio of 3 to 2. By farther exhausting the air, so as to make its density correspond with ten inches of the column of quicksilver, the mean of a like number of days gave a remarkably steady rate of  $+1''.7$ , being an increase of  $3''.0$  on the former rate ; and by continuing the exhaustion until the mercurial column sunk to an inch, the average of the same number of days produced a rate of  $+6''.6$ , being a farther increase of  $4''.9$ . Thus, an alteration of  $+9''.7$  in the rate of the chronometer was produced by diminishing the density of the air in the ratio of 30 to 1 ; and on removing it from the

\* That chronometers, having their balances magnetic, experience alterations from being placed in different positions with respect to the magnetic meridian, is no longer doubtful.

highly rarefied air in which the last experiment placed it, to the full pressure (29.85 inches) of the atmosphere, the mean of ten days observations gave an average rate of  $-6''.3$ ; the time-keeper having altered its detached rate  $-3''.2$ , in consequence of the experiment. These alterations of rate were produced in each case immediately after the density of the air was changed.

Adopting the last mentioned rate as a standard, the time-keeper was next submitted, during four days, to a pressure denoted by 25 inches of the mercurial column, when its rate was immediately changed from  $-6''.3$  to  $-2''.2$ , being an increment of  $4''.1$ . By farther diminishing the density to a quantity represented by 10 inches of quicksilver, the average of a like period gave a rate of  $+2''.7$ , being another increment of  $4''.9$ ; and on again exhausting the air till the mercury was depressed to 5 inches, the mean of four days produced a rate of  $+4''.6$ , being a third increment of  $1''.9$ ; and by restoring the chronometer to the ordinary pressure of the atmosphere, an immediate change in its rate took place; the mean of ten days producing an average of  $-5''.9$ , the close approximation of which to its former detached rate being remarkable.

The results of the preceding experiments are entered for the purpose of a more convenient reference in the succeeding tables; together with the average heights of the barometer during the times the detached rates were determined; and also the mean temperatures through the entire series of observations.

*on the rates of chronometers.*

Eight-day Box Chronometer A.			
Mean Temp.	Pressure.	Number of days.	Mean daily rate.
First set of Experiments.			
58	detached 30 in.	5	- 3".1
59	20 in.	4	- 1".3
59	10 in.	4	+ 1".7
59	1 in.	4	+ 6".6
59	detached 29.85 in.	10	- 6".3
Second set of Experiments.			
59	detached 29.85	10	- 6".3
57	25 in.		- 2".2
58	10 in.	4	+ 2".7
58	5 in.	4	+ 4".6
57	detached 30.02 in.	10	- 5".9

These remarkable changes having taken place in the daily rate of a chronometer, known under the ordinary circumstances of atmospheric pressure to preserve a steady uniformity of rate, I was induced to employ, in the next place, three good pocket chronometers, whose detached rates are

recorded in the first horizontal line of the succeeding table. The time-keepers were placed at the same instant beneath the receiver, and the air exhausted until the mercurial column sunk to half an inch; when the singularly large increments entered in the second line were produced. In the third horizontal line of the table, the rates of the chronometers are given after they were restored to the full pressure of the atmosphere; and it is remarkable, how closely they approximate to the primitive rates, the greatest aberration being only 1".3, in chronometer C. Nor ought the close approximation also of the large increments produced by the exhaustion to a state of equality to be entirely overlooked. The chronometers were taken each day from beneath the receiver for the purpose of comparison, and being wound up; a circumstance only necessary once a week, with the chronometer A.

Experiments with Pocket Chronometers, B, C, D.					
Mean Temp.	Pressure.	Number of Days.	Mean daily rate of B.	Mean daily rate of C.	Mean daily rate of D.
58°	detached 30.04 in.	6	+ 4".7	+ 0".3	+ 3".6
58°	$\frac{1}{2}$ inch	6	+ 23".5	+ 18".6	+ 23".5
56°	detached 30.01 in.	6	+ 5".0	+ 1".6	+ 2".4

The preceding changes having been produced by diminishing the density of the air in the ratio of 60 to 1; an

attempt was next made, to ascertain the effect of a more gradual diminution thereof, on the same chronometers. The results are recorded in the following table :

Experiments with Pocket Chronometers, B, C, D.

Mean Temp.	Pressure.	Number of Days.	Mean daily rate of B.	Mean daily rate of C.	Mean daily rate of D.
56°	detached 30.01 in.	6	+ 5".0	+ 1".6	+ 2".4
58°	20 in.	4	+ 9".3	+ 6".2	+ 9".3
58°	10 in.	4	+ 18".5	+ 11".0	+ 17".6
57°	detached 29.76 in.	4	+ 6".6	+ 2".1	+ 3".5

A still more gradual decrease of density was preserved in another set of experiments, the results of which are recorded in the next table.

Experiments with Pocket Chronometers, B, C, D.

Mean Temp.	Pressure.	Number of Days.	Mean daily rate of B.	Mean daily rate of C.	Mean daily rate of D.
57°	detached 29.76 in.	4	+ 6".6	+ 2".1	+ 3".5
59°	25 in.	4	+ 10".4	+ 3".6	+ 5".7
59°	15 in.	4	+ 12".2	+ 7".6	+ 12'.1
60°	5 in.	4	+ 19".2	+ 11".4	+ 20".6
60°	detached 29.84 in.	4	+ 6".0	+ 0".6	+ 4".2

It is remarkable, that the increments resulting from the unequal detached rates of the chronometers B and D, should in two instances, viz. those corresponding to half an inch, and 20 inches of quicksilver, be precisely the same; and in three other cases, viz. 15 inches, 10 inches, and 5 inches respectively, very nearly so; the only considerable deviation being in the rate of the time-keeper D, when under a pressure corresponding to 25 inches of quicksilver. This chronometer however recovered itself when under a pressure of 15 inches.

In another set of experiments, and of which the results are found in the succeeding table, the density of the air under the receiver was uniformly diminished by decrements represented by two inches of quicksilver, and which was accompanied by changes in the rates of the two chronometers employed, (abstracting the occasional aberrations displayed by most time-keepers) increasing proportionally as the density of the air was decreased. From the nearly equal uniformity of temperature also that prevailed during each set of experiments, and from the positions of the chronometers with respect to the magnetic meridian having likewise been preserved constant, there can be no doubt but the different alterations of rate are due to alterations of pressure.

Experiments with Pocket Chronometers C, D.				
Mean Temp.	Pressure.	Number of Days.	Mean daily rate of C.	Mean daily rate of D.
46	Detached 29.1 in.	4	+ 2.5	+ 4.0
46	27	4	+ 3.0	+ 5.2
47	25	4	+ 3.4	+ 6.1
46	23	4	+ 4.7	+ 7.2
45	21	3	+ 6.4	+ 9.4
42	19	4	+ 7.6	+ 11.3
43	17	3	+ 10.5	+ 13.4
44	15	4	+ 11.7	+ 14.4
45	13	4	+ 13.4	+ 15.8
49	11	4	+ 14.5	+ 17.1
48	9	4	+ 16.0	+ 18.2
48	7	3	+ 19.0	+ 20.4
50	5	4	+ 18.6	+ 22.1
49	3	4	+ 19.9	—

The next set of experiments was performed with the eight day chronometer E, possessing the very excellent detached rate recorded in the first of the following columns. In the second column of the same table is also entered the daily rate of the same chronometer when placed in air, diminished in density in the ratio of 60 to 1; and in the third column is its rate, when afterwards restored to the full pressure of the atmosphere. The mean temperature of the first period was 56°, of the second 58°, and of the third 57°.



Experiments with an Eight-day box Chronometer, E.		
Mean Pressure prior to the Experiment 29.95 inches.	Pressure during the Experiment half inch.	Mean Pressure after the Experiment 29.80 inches.
Daily Rates.	Daily Rates.	Daily Rates.
— 5.4	— 7.5	— 6.3
— 5.1	— 8.7	— 7.1
— 5.6	— 9.5	— 5.8
— 5.8	— 9.2	— 5.8
— 5.7	— 9.2	— 5.1
— 5.6	— 9.0	— 5.9
— 5.2	— 9.4	— 6.0
— 5.1	— 9.4	— 6.6
— 5.5	— 9.7	— 5.0
— 5.6	— 9.9	— 5.7
— 5.4	— 9.9	— 5.7
Mean daily rate. — 5.45	Mean daily rate. — 9.32	Mean daily rate. — 5.91

The preceding chronometer having therefore *decreased* its rate in consequence of a *diminution* of pressure, contrary to the uniform character of the former experiments, I was induced to extend my field of investigation; and for that purpose obtained from my naval friends several more chronometers. And in order to render the experiments of greater utility in their ultimate applications, I decided on submitting the time-keepers to the influence of air corres-

ponding in density to the mean state of the atmosphere, at some remarkable places, situated at considerable elevations above the level of the sea.

For this purpose several chronometers were supposed to be carefully transported from London to Geneva, every circumstance relating to temperature and magnetism remaining constant; and from the succeeding table it will appear, that an alteration in the rate of the time keeper will be the necessary result. In this experiment two box chronometers, and three of the pocket kind, had their rates carefully determined for five days, under a mean atmospheric pressure of 29.86 inches, and an average temperature of 50°; and for a like number of days in air corresponding in density to 28.6 inches, being the mean atmospheric pressure at Geneva, the average temperature being 48°. The following alterations of rate resulted; and which clearly demonstrated that a constant difference of 1.26 inches in the mercurial columns, was capable of affecting very sensibly the rates of the chronometers employed.

Experiment for Geneva, elevated 201 fathoms above the level of the Sea.								
Place.	Mean Temperature during the Experiment.	Mean Pressure.	Number of days.	Mean daily rate of A.	Mean daily rate of C.	Mean daily rate of F.	Mean daily rate of G.	Mean daily rate of H.
London,	50	Detached 29.86 in.	5	- 3.3	+ 0.6	+ 2.0	- 7.1	+ 7.9
Geneva,	48	28.60 in.	5	- 2.7	+ 1.2	+ 0.5	- 3.1	+ 9.7

Of the above alterations of rate it will be perceived, that the box chronometer F *lost* by being submitted to a *diminished* pressure; but that the others *gained*.

These results having been produced by air corresponding in density to its mean state at 201 fathoms above the level of the sea, another set of experiments was instituted on the supposition that a chronometer was removed from the shores of the Mediterranean to the lofty plains of La Mancha and the Castiles, where the mean barometric pressure is denoted by 27.81 inches. The following table contains the results :

Experiment for Madrid, elevated 329 fathoms above the level of the Sea.							
Place.	Mean Temp. during the Experiment.	Mean Pressure.	Number of Days.	Mean daily rate of A.	Mean daily rate of F.	Mean daily rate of H.	Mean daily rate of I.
London	52	detached 29.86 in.	4	— 4.5	+ 3.1	+ 7.4	— 17.4
Madrid	53	27.81	5	— 2.4	+ 0.8	+ 9.6	— 13.1

Three of the chronometers employed in the two preceding sets of experiments, viz., A, F, H, were the same, and which therefore enables us to perceive, that a difference of  $\frac{8}{10}$ ths of an inch in the barometric column produces an alteration of rate. The changes also are more considerable in proportion as the difference in the altitudes of the mercurial column is increased, as is proved in the succeeding table.

Difference in the Altitudes of the Mercurial Columns.	Increment communicated to the Chronometer A.	Increment communicated to the Chronometer H.	Decrement communicated to the Chronometer F.
1.26 in.	+ 0".6	+ 1".8	— 1'.5
.205 in.	+ 2".1	+ 2".2	— 2".3

The mean also of five days observations with chronometer H, in a medium corresponding in density to 25.95 inches of mercury, being the average state of the barometer at the Palace of Ildefonso, 630 fathoms above the ocean, produced an increment in its rate amounting to 3".0; and which taken in conjunction with the results of the same chronometer recorded in the preceding table, prove that it's rate varied nearly with the density of the medium in which it was placed. This chronometer is one, on which in many delicate inquiries I particularly depended.

Although it appears from the preceding experiments, that the alterations of rate in the *same* chronometer depend on the density of the medium in which it is placed; yet the magnitudes of the changes in *different* time-keepers appear to be very unequal, arising no doubt from peculiarities of construction. Thus, a set of experiments was undertaken with two good box chronometers, in order to discover what changes would take place in their rates if they were transported from Vera Cruz, on the shores of the Pacific Ocean, to the Table Land of Mexico, where the mean atmospheric pressure is denoted by 23 inches of the barometer. The result proved, that the change in the rate of the chronometer represented by K, amounted only to + 1".9, and in the time-keeper L, to - 5".0; the former having *gained* by the *diminished* pressure, and the latter *lost*.

Experiment for Mexico, elevated 1253 fathoms above the level of the Sea.					
Place.	Mean Temp. during the Experiment.	Mean Pressure.	Number of Days.	Daily rate of K.	Daily rate of L.
Vera Cruz	54°	30 in.	5	— 5".9	+ 4".8
Mexico	54°	23 in.	4	— 4".0	— 0".2

If we again conceive a set of chronometers removed from the level of the ocean to Santa Fé de Bogota, or to the still loftier city of Quito, at which latter place the mean altitude of the mercurial column is 21 inches, the alterations of rate, recorded in the next table, will be found to result from the diminished pressure.

Experiment for Quito, elevated 1603 fathoms above the level of the Sea.					
Place.	Mean. Temp. during the Experiment.	Mean Pressure.	Number of Days.	Daily rate of H.	Daily rate of C.
Level of the ocean.	48°	30 in.	4	+ 8".0	+ 3".0
Quito	47°	21 in.	5	+ 14".2	+ 6".2

Four chronometers were afterwards placed in air corresponding in density to 15 inches of mercury, being nearly the altitude of the barometer on Chimborazo, when the results recorded in the following table were produced.

Experiment for the summit of Chimborazo, elevated 3573 fathoms above the level of the Sea.							
Place.	Mean Temp. during the Experiment.	Mean Pressure.	Number of days.	Daily rate of A.	Daily rate of C.	Daily rate of F.	Daily rate of H.
Level of the Sea	52°	30 in.	4	— 1".9	+ 2".6	+ 2".0	+ 7".9
Summit of Chimborazo	51°	15 in.	4	+ 4".2	+ 9".6	— 3".3	+ 17".0

Three of the chronometers used in the last experiment, viz. A, F, and H, were the same as those employed in the experiments for Geneva and Madrid; and from which it will be perceived, that even under the influence of a much less pressure, the alterations of rate partook of the same character; in one instance (F) *losing* from *diminished* pressure, and in the other cases (A, H,) *gaining*. The changes in the last experiment are more considerable, in consequence of the greater rarity of the air.

As a final experiment on this part of the subject, the two excellent pocket chronometers (C, H,) employed in the last experiment, were placed beneath the receiver, and the air exhausted until the mercurial guage sunk to 12.95 inches, being the elevation of the barometric column observed by GAY LUSSAC in his magnificent aerostatic ascent. The consequent alterations of rate were respectively + 13".2, and + 19".2.

From the preceding experiments it may therefore be inferred, that a chronometer constructed in air, corresponding in density to its mean state at the level of the ocean, will undergo alterations of rate, when removed into a region

where the average density is sensibly diminished. In a subsequent part of the paper it will also appear, that the uncertain fluctuations of the barometer at the level of the sea, produces in many cases minute, but sensible changes of rate.

2. The effects recorded in the preceding section of the paper having been produced by a *diminution* of atmospheric pressure, it was conceived, that results entirely the reverse would arise from an increase thereof; that is, if a chronometer *gained* by being placed in air of a *less* density than that afforded by the ordinary state of the atmosphere, it ought to *lose*, by being subjected to air of a *greater*. Accordingly, by introducing different time-keepers into a condensing engine, furnished with an appropriate mercurial guage, the *opposite results* here alluded to actually took place.

Not knowing the effects likely to be produced on so delicate a machine as a chronometer, by placing it in air condensed to any considerable degree, the time-keeper H, employed in the former investigations, was first placed in air of a density corresponding to 34 inches of mercury. The result was an alteration of the mean detached rate of the chronometer from  $+6''.9$  to  $+5''.3$ . The chronometer was necessarily removed from the condensing machine for a few minutes each day, for the purpose of comparison, and being wound up.

On restoring the time-keeper to the ordinary pressure of the atmosphere, its rate returned to  $+6''.5$ , agreeing within  $0''.4$  of its former detached rate. In a second experiment, it was subjected to air denoted in density by 38 inches of quick-silver, when its rate altered from  $+6''.5$  to  $+2''.3$ ; and on

being again detached, its rate was restored to  $+ 6''.6$ , agreeing within  $0''.1$  of its former detached rate. These mean rates were determined in each case for five days. Hence, by contrasting the results obtained under the receiver of the air pump with those produced in the condensing engine, it will appear, that the chronometer *gained* in each experiment by *diminishing* the density of the air, and, on the contrary, *lost*, by *increasing* it.

In the next experiment the box chronometer M was placed in the condensing engine, under pressures corresponding to the mercurial columns recorded in the following table; and from which resulted an alteration of rate from  $- 6''.9$  to  $- 16''.1$ .

Experiments with Box Chronometer M.			
Mean Temp.	Mean Pressure.	Number of days.	Mean daily rate of M.
49°	30 in.	5	$- 6''.9$
48	36 in.	5	$- 8.3$
48	42 in.	6	$- 9.5$
47	48 in.	5	$- 10.8$
48	54 in.	6	$- 16.1$

This chronometer, when placed under the receiver of the air pump in air of a less density than the ordinary state of the



atmosphere, always received *increments* to its rate ; and from its having *lost* through the whole of its experiments relating to the condensed air, the same inference may be deduced from it as from the preceding chronometer. The decrements in the above table, it will be remarked, are very nearly uniform, from 30 inches to 48 inches ; but in the transition from 48 inches to 54 inches, the change is considerable. It is also worthy of observation, that this time-keeper, when finally detached for twenty days, preserved a rate of nearly  $-15''.0$  ; the dense air to which it had been so long exposed having materially augmented its losing rate, and apparently, communicated to it a permanent character.

The third set of experiments was with the pocket chronometers B and C, employed in the inquiries with the air pump, and the results of which are entered in the succeeding table.

Experiments with the Pocket Chronometers B, C.				
Mean Temp.	Mean Pressure.	Number of days.	Mean daily rate of B.	Mean daily rate of C.
47 <sup>o</sup>	30 in.	6	+ 4.3	+ 1.2
47	45 in.	5	- 5.2	- 4.4
46	60 in.	5	- 9.7	- 8.2
45	75 in.	5	- 17.2	- 9.5
46	30 in.	5	+ 4.5	+ 0.6

The rates recorded in the above table, it will be perceived,

are *decrements*; whereas, in the experiments performed in the rarified air with the same chronometers, they were uniformly found to be *increments*; and hence, the results agree precisely with those recorded respecting the time-keepers H and M. The almost perfect restoration of the detached rates, after the great changes produced by so considerable an augmentation of density as that corresponding to the mercurial column of 75 inches, is a very remarkable feature of the table.

3. To obtain alterations of rate of the most striking and remarkable kind, the effects of suddenly removing chronometers from condensed into rarefied air, and *vice versa*, were estimated by a series of careful experiments.

A box chronometer, N, having its detached rate exactly coinciding with mean time when the barometric column was 29.95 inches, on being placed in air of a double density altered its rate to  $-8''.6$ ; and when afterwards placed under the receiver in air corresponding in density to an inch of mercury, the daily average became  $+10''.7$ ; the difference in the densities having produced an alteration of  $19''.3$  in the rate. The observations were continued for each experiment five days; and the changes in the rates were produced *immediately* after the time-keeper was removed from one condition to the other. In another experiment with the same chronometer, the mean of six days, under a pressure denoted by 15 inches of quicksilver, gave a result of  $+4''.1$ ; but on placing it in air, having three times the mean density of the atmosphere, the average of the same number of days was  $-16''.7$ . Hence it appears, that by removing the time-

keeper from condensed into rarefied air, it *gained*; and conversely, from rarefied into condensed air, it *lost*.

In a third experiment, the pocket chronometer O, gave a mean rate of  $-6''.5$  when placed in air of a density denoted by 26 inches of the mercurial column; but when placed in the condensing engine, in air of a density denoted by 45 inches of quicksilver, the rate changed to  $-2''.7$ ; and on afterwards restoring it to the ordinary pressure of the atmosphere during four days, the average rate became  $-7''.7$ . The character of this chronometer, during many preceding experiments, was to *gain* with *greater* pressure, and to *lose* with *less*; and the preceding experiment perfectly accords with the same results.

In a fourth experiment, the pocket chronometer C was employed, having a mean rate of  $+1''.2$  when placed for six days under the receiver in air corresponding in density to 28 inches of mercury; but on introducing the time-keeper into the condensing engine, in air of a density denoted by 60 inches of quicksilver, the rate immediately declined to  $-8''.2$ . In a converse experiment with the same chronometer, the rate was first determined in the condenser when the density corresponded to 36 inches of mercury, the average being  $+0''.7$ ; but on removing it into the receiver, and exhausting the air until the mercurial gauge sunk to 27 inches; the mean of four days observations gave an augmented rate of  $+3''.9$ ; and on restoring the chronometer to air of the ordinary density, its mean rate became  $+2''.6$ , agreeing within half a second of its original detached rate. This time-keeper, in many antecedent experiments, had always been

found to *gain* with *less* pressure, and *lose* with *more*; and the tenor of the last experiments confirms the same law.

The preceding results for the chronometers C and N, may be conveniently arranged in the following tables.

Effects on the Chronometers C and N, when removed from condensed into rarefied air.							
Mean Temp.	Mean Pressure.	Number of days.	Mean daily rate of C.	Mean Temp.	Mean Pressure.	Number of days.	Mean daily rate of N.
50°	36 in.	5	+ 0".7	47°	60 in.	5	- 8".6
48°	27 in.	4	+ 3".9	46°	1 in.	5	+ 10".7

Effects on the Chronometers C and N, when removed from rarefied into condensed air.							
Mean Temp.	Mean Pressure.	Number of days.	Mean daily rate of C.	Mean Temp.	Mean Pressure.	Number of days.	Mean daily rate of N.
49°	28 in.	6	+ 1".2	46°	15 in.	6	+ 4".1
51°	60 in.	5	- 8".2	45°	90 in.	6	+ 16".7

An opportunity in another experiment was taken, when the chronometers C and D had been under the diminished pressure of 23 inches for five days, to remove them for a like period into the condensing engine, containing air of double the mean density of the atmosphere. The result of this application was, that the former rate became a decrement of

9".5, and the latter a rate of a similar kind of 10".4. Knowing the merits of these chronometers, I ventured to predict, that if they were removed from the condensed air into an atmosphere corresponding in density to 21 inches of quicksilver, the transition would produce rates *greater* than those corresponding to 23 inches, in consequence of the time-keeper being placed by such an experiment in air of a less density than that corresponding to the first experiment. The result verified the conjecture; the average rate of the time-keeper C having become + 5".0, and that of Chronometer D + 12".0. These interesting results are recorded in the next table.

Experiments with the chronometers C and D, when removed from rarefied into condensed air, and <i>vice versa</i> .				
Mean Temp.	Mean Pressure.	Number of Days.	Mean daily rate of C.	Mean daily rate of D.
47°	23 in.	5	+ 2' 5.	+ 9".4
49°	60 in.	5	- 7".0	- 1".0
48°	21 in.	5	+ 5".0	+ 12".0

By contrasting also the rates of the chronometers B and C, when subjected, as in two of the preceding experiments, to air corresponding in density respectively to half an inch, and 75 inches of quicksilver; it will be perceived, that the effect of these opposite densities was to occasion an alteration of 40".7 in the rate of the time-keeper B, and of 28".1 in that of C. These results are recorded in the next table.

Experiments with the Chronometers B and C, when removed from rarefied into condensed air.		
Pressure.	Mean daily rate of B.	Mean daily rate of C.
$\frac{1}{2}$ in.	+ 23".5	+ 18".6
75 in.	- 17".2	- 9".5

4. It becomes now an interesting enquiry, to consider how far the ordinary changes in the density of the air may be likely to exercise an influence on the rate of a chronometer. The range of the mercurial column in London may, on an average, be estimated at  $2\frac{3}{4}$  inches; and there can be no doubt but the difference produced in the density of the air by such a range must, if the transition be at all sudden, and the difference of density constant for twenty-four hours, or even less, be sufficiently considerable to affect the majority of chronometers. A great difference, however, appears to exist in this respect among time-keepers. The change of density, that in one machine of this kind would occasion an alteration or rate amounting to several seconds, in another, would scarcely produce any sensible effects; and I have found, during the whole of these experiments, a considerable difference in this particular between pocket and box chronometers; the former being most readily affected by alterations of atmospheric density.

In the following tables are recorded the results of different experiments, instituted with a view of determining the

alterations of rate occasioned by small depressions of the mercurial column below the average state of the barometer for some days previous to each experiment.

Changes produced in the rates of the Chronometers H, O, P, Q, in consequence of a diminution of pressure denoted by one inch of mercury.

Mean Temp.	Mean Pressure.	Mean daily rate of H.	Mean Temp.	Mean Pressure.	Mean daily rate of O.	Mean Temp.	Mean Pressure.	Mean daily rate of P.	Mean Temp.	Mean Pressure.	Mean daily rate of Q.
47	30.3	+ 8.5	47	30.1	+ 8.0	45	30.1	- 7.0	49	29.9	+ 4.2
46	29.3	+ 9.3	48	29.1	+ 8.7	46	29.1	- 7.9	50	28.9	+ 3.4

The chronometer H was employed in many of the former experiments, and it will be perceived that it constantly *gained* with *less* pressure. In the investigations instituted for Geneva and Madrid, for example, the increments communicated to its rate were respectively 1".8, and 2".2; and in the present instance an aberration of the same kind was produced, amounting to 0".8; the increment being smaller than either of the preceding, in consequence of a less diminution of pressure. The close approach to numerical equality also in the changes of rates recorded in the preceding table, is not unworthy of notice, the differences being respectively - 0".8, + 0".7, - 0".9, and + 0".8. The time devoted to each set of experiments was three days.

In another set of experiments, the pocket chronometer D, and the box chronometer F, were again resorted to, and placed under the different pressures recorded in the succeeding table. The mean temperatures during the several periods of observation underwent no very considerable

change; and from the uniformly decreasing nature of the rates it is fair to infer, that the alterations resulted from diminished pressure alone, and that the successive inches of quicksilver were capable of producing them. It is worthy of observation also, that the *same* pressure which produced an *increment* in the rate of the former chronometer, occasioned a *decrement* in that of the latter; and by a reference to several of the foregoing experiments, it will be found that the time-keeper D in all cases *gained* with *less* pressure, and the chronometer F the contrary.

Changes produced in the rates of the Chronometers D, F, in consequence of regular decrements in the pressure denoted by an inch of mercury.							
Mean Temp.	Mean Pressure.	No. of Days.	Mean daily rate of D.	Mean Temp.	Mean Pressure.	No. of Days.	Mean daily rate of F.
51	30.4	5	+ 4.2	51	30.4	5	+ 3.9
50	29.4	4	+ 6.4	50	29.4	4	+ 3.1
50	28.4	4	+ 7.4	50	28.4	4	+ 2.5
48	27.4	4	+ 8.0	48	27.4	4	+ 2.1

In the next set of experiments, the time-keepers A, C, F, and R were employed. The results are recorded in the next table; and it will be perceived that a difference of pressure amounting only to 0.69 inches of mercury, for twenty-four hours, was capable of producing an alteration in the rate of F, of the same kind as observed in the experiments with



the same chronometer for Geneva, Madrid, Chimborazo, &c. The time-keeper R likewise lost with a diminution of pressure. The chronometers A and C received increments to their rates, in perfect conformity to those obtained in former experiments.

Changes produced in the rates of the chronometers A, C, F, R, in consequence of small but irregular decrements in the pressure.

Mean Temp.	Mean Pressure.	No. of days.	Mean rate of A.	Mean rate of C.	Mean rate of F.	Mean rate of R.	Diff. of pressure.	Change of rate in A.	Change of rate in C.	Change of rate in F.	Change of rate in R.
	Inches.		"	"	"	"	Inches.				
47	29.26	4	-2.9	+3.4	+2.5	-6.5	-0.96	+1.4	+0.5	-1.2	-0.8
49	28.30	4	-1.5	+3.9	+1.3	-7.3	+1.38	-3.2	-1.8	+0.3	+0.9
50	29.68	4	-4.7	+2.1	+1.6	-6.4	-1.68	+0.6	+0.8	-1.2	-1.1
48	28.00	4	-4.1	+2.9	+0.4	-7.5	+1.69	-	-	+0.8	-
46	29.69	4	-	-	+1.2	-	-0.69	-	-	-1.7	-
42	29.00	4	-	-	-0.5	-	-	-	-	-	-

To throw as much light as possible on this very interesting and important part of the investigation, six distinct sets of experiments were performed with the pocket chronometer H, and which, as before remarked, has been found uniformly to maintain an excellent rate when under a constant atmospheric pressure; and to display an immediate alteration whenever the density of the air was sensibly changed. In the succeeding table, it will be observed by inspecting the fifth and sixth columns, that in every case the rate of the time-keeper was *increased* as the density was *diminished*, and *vice versa*; and also, that an alteration of 0.6 inches in the mercurial column produced a sensible alteration in the time-keeper. The want of perfect proportionality in the rates,

may be attributed to the minute aberrations of the chronometer, produced by imperfections of workmanship, and to the errors arising from observation.

Changes produced in the rate of the time-keeper H, by small but irregular decrements in the pressure.					
Mean Temp.	Mean Pressure.	Number of Days.	Mean daily rate.	Difference of Pressure.	Change of rate.
o	inches.			Inches.	
51	29.26	4	+ 8.7	- 0.96	+ " 1.0
49	28.30	4	+ 9.7	+ 1.38	- 0.4
47	29. 8	4	+ 9.3	- 1.68	+ 1.4
46	28.00	4	+ 10.7	+ 1.69	- 1.3
45	29.69	4	+ 9.4	- 1.69	+ 1.2
48	28.00	4	+ 10.6	+ 2.25	+ 1.2
47	30.25	4	+ 9.4	- 1.25	+ 0.4
44	29.00	4	+ 9.8	+ 1.02	- 0.3
43	30.00	4	+ 9.5	- 0.77	+ 0.5
42	29.25	4	+ 10.0	+ 0.60	- 0.4
40	29.85	4	+ 9.6	- 0.66	+ 0.6
40	29.25	4	+ 10.2		

From these experiments it may be therefore inferred, that a difference in the density of the atmosphere, represented by a quantity less than an inch of quicksilver, if continued for a day, was capable of affecting all the chronometers employed; and this is an atmospheric change by no means uncommon in this variable climate. Nor is it indeed neces-

sary that the alteration of density should even continue for twenty-four hours, since, from the change of rate being instantaneous (as will be proved in a subsequent page) six hours will be sufficient, in some cases, to disclose it. In cases however where the variations of the mercurial column are but small, and its transition from one state to another marked by a gradual character, the effect on the generality of chronometers is scarcely, if at all perceptible.

With a difference in the mercurial column of an inch and three quarters, or two inches, I have little doubt but all time-keepers will be influenced; and it is moreover known, that from a species of reaction in the atmospherical columns, it not unfrequently happens that the greatest depression of the barometer succeeds to a considerable elevation of it, and *vice versa*, so as to exhibit a difference of this kind. In the instance of the remarkable depression of the barometer, in December 1821, Mr. HOWARD informs us it sunk on the 25th instant to 27.83 inches, and on the 27th remained for 12 hours stationary at 28.07 inches; and from which time to the 31st it rose to 30 inches. Now, many examples might be selected from the experiments recorded in the preceding pages, to prove that a difference of two inches in the barometer, for twelve hours, would be sufficient to produce an alteration of rate; and there can be little doubt, that had the rates of some good chronometers been carefully attended to\* during this singular alteration of atmospheric density, variations of rate, at least equivalent to that produced by

\* I have attempted, but without success, to obtain the rates of some good chronometers during this period.

transporting a time-keeper from London to Geneva, would have been observed.

The sudden changes to which the density of the atmosphere is sometimes liable in this climate, renders it necessary, therefore, that a correction should be applied to the rate of a chronometer, proportional to the alteration of density; the correction partaking in some cases of a positive character, and in others of a negative. A similar correction must likewise be necessary when a traveller ascends to any considerable elevation above the sea; for example, to Geneva, to the plains of the Castiles, or to the table land of Mexico. The value of the correction will be different for different time-keepers, and in all cases must be determined by previous experiment.

The changes here alluded to can influence chronometers only beyond the tropics, since between them, it is known that the fluctuations of the barometer do not much exceed a quarter of an inch; but in the arctic regions, where the causes which promote alterations of atmospheric density are the greatest, the effect on the time-keeper must be the greatest also.

In proportion however as we ascend above the level of the sea, the uncertain changes of the barometer are known to approximate to uniformity; and therefore at higher elevations, the same chronometer would preserve a greater regularity of rate than in the lower regions of the air.

It becomes now an interesting question to determine, if the alterations of rate displayed by the different chronometers, under the various circumstances in which they have been placed in the preceding experiments, is *immediately* acquired the

moment the change of pressure takes place ; or whether it is an effect which the air gradually produces on the machine.

By a reference to the experiments performed with the box chronometer E, it would seem as if the alteration of pressure required two days to produce its full effect on the rate ; but from other experiments now about to be recorded, and on which I place a greater dependence, it would appear that the change is *immediate*.

A pocket and box chronometer, possessing detached rates of  $+ 9''.0$ , and  $+ 1''.9$ , were placed under the receiver in air denoted in density by 2 inches of the mercurial column ; and which great degree of exhaustion was employed in order that, by producing considerable alterations of rate, the changes during very small intervals of time might be perceptible.

At the expiration of an hour, the increment produced in the rate of the pocket chronometer by a mean of three observations was found to be  $+ 1''.33$  ; whereas the detached rate in the same time would have amounted only to  $+ 0''.37$ , being a clear increase of  $0''.96$  in consequence of the diminished pressure. At the end of the second hour the mean rate was found to be  $+ 1''.23$  ; and in like manner at the termination of the third  $+ 1''.35$  ; of the fourth  $+ 1''.30$  ; of the fifth  $+ 1''.10$  ; of the sixth  $+ 0''.80$ , and at which rate it continued for several hours. At the end of the nineteenth hour the rate recovered itself and became  $+ 1''.28$  ; at the twenty-second hour  $+ 1''.25$ , the twenty-third  $+ 0''.80$ , and at the twenty-fourth  $+ 1''.05$ . These different results may, in a practical point of view, be regarded as uniform, considering the unavoidable errors of observation in attempting to

estimate the exact value of such minute inequalities. Indeed, the mean of the hourly observations from noon to midnight presented the same result as the mean from midnight to noon, the former being  $+ 1''.12$ , and the latter  $+ 1''.10$ . The entire rate for the twenty-four hours amounted to  $+ 26''.6$ , being an increase on its detached rate of  $17''.6$ . In like manner, the mean of three comparisons for the first of the horary observations with the box chronometer presented a rate of  $+ 6''.76$ ; whereas the detached rate during the same time would have been  $+ 0''.08$ , exhibiting an increment of  $0''.68$  due to the diminished density of the air. By continuing the horary observations during the twenty-four hours, it was found that the mean of the horary rates for the first twelve hours was  $+ 0''.92$ , and of the last  $+ 0''.72$ . The entire rate for the whole period was  $+ 20''.4$ , being an increment to its detached rate of  $18''.5$ .

The succeeding day the two chronometers were restored to the full pressure of the atmosphere; and the first hour after their restoration, an attempt was made to discover the same increment in the rate of the pocket chronometer as that which it possessed under the receiver, but without effect; the mean of four comparisons giving only a rate of  $+ 0''.45$ , an increment bearing evidently a relation to the primitive detached rate of the time-keeper. At the end of three hours the mean rate per hour was found to be  $+ 0''.40$ ; at the end of six hours  $+ 0''.46$ ; at the expiration of nine hours  $+ 0''.41$ ; at the end of eighteen hours  $+ 0''.39$ ; and at the end of twenty-four hours  $+ 0''.44$ ; the mean of the whole being  $+ 0''.42$ , and producing a rate for the entire twenty-four hours of  $+ 10''.08$ . Similar observations with the box

chronometers produce a rate agreeing exceedingly near with mean time; the entire rate for twenty-four hours being  $+1''.0$ , agreeing very nearly with its former detached rate.

Hence it may be inferred, that the change produced in the rate of a chronometer by *diminishing* the density of the air, is *immediate* and *uniform* in its effects; and so also is the effect produced by *increasing* it.

It may not be uninteresting to furnish a few examples illustrative of the power which most chronometers have of regaining their original rates, or very nearly so, after they have been subjected to pressures, both considerably above and below the mean density of the air; a property by which they are enabled to recover any temporary derangement they may undergo. The following instances are selected from the time keepers placed under the receiver of the air pump. The detached rates are those obtained under the ordinary circumstances of the atmosphere. The pressure to which the chronometers were subjected, are denoted by the several inches of quicksilver.

Examples of Chronometers having nearly regained their original rates, after being subjected to different Pressures under the Receiver.			
Detached $+ 4.9$ 3 in. $- 23.8$ Detached $+ 2.5$	Detached $- 6.3$ 5 in. $+ 4.3$ Detached $- 5.9$	Detached $+ 7.5$ 5 in. $+ 26.9$ Detached $+ 8.7$	Detached $+ 4.4$ 5 in. $+ 21.2$ Detached $+ 3.4$
Detached $+ 7.9$ 12 in. $\div 19.2$ Detached $+ 8.2$	Detached $- 1.9$ 15 in. $+ 4.2$ Detached $- 2.6$	Detached $- 5.9$ 20 in. $- 4.0$ Detached $- 6.4$	Detached $+ 6.5$ 26 in. $+ 9.5$ Detached $+ 7.3$

The following are examples derived from the experiments performed with the condenser.

Examples of Chronometers having nearly regained their original rates, after being subjected to different Pressures in the Condensing Engine.				
Detached + $7.3$ 40 in. + $3.1$	Detached + $4.3$ 75 in. - $17.2$	Detached + $0.2$ 75 in. - $9.5$	Detached + $7.5$ 90 in. - $2.2$	Detached + $4.4$ 90 in. - $5.0$
Detached + $7.4$	Detached + $4.5$	Detached - $0.6$	Detached + $8.7$	Detached + $3.4$

A similar restoration of rate was found also to take place by removing a chronometer from rarefied to condensed air, and afterwards restoring it to air of the previous density, as exemplified in two instances in the next table.

Examples of Chronometers having nearly regained their original rates, after being removed from rarefied to condensed air.			
Mean Pressure.	Mean daily rate.	Mean Pressure.	Mean daily rate.
23 inches.	+ $11''.6$	23 inches.	+ $5''.5$
90 inches.	+ $1.6$	90 inches.	- $4.4$
23 inches.	+ $12.8$	23 inches.	+ $6.1$

It was also found, that by restoring the above chronometers to air of any other density than that from which they were originally taken, the rate would not return to its primitive quantity. When, for example, they were placed under a pressure of 21 inches, the rate of the first chronometer became +  $14''.2$  instead of +  $11''.6$ ; and the daily increase of the second +  $7''.2$ , instead of +  $5''.5$ ; results in exact conformity with the general law which regulated the same chronometers in other experiments.



As a striking example of the power possessed by a chronometer, of immediately altering its rate to a quantity corresponding with every new circumstance in which it was placed, and also of regaining its original rate, after it was again restored to its primitive condition, the following table may be referred to; and in which it will be perceived, that the original rate with which the chronometer departed, was still possessed within the small quantity 0".2, after the last experiments. During the course of observations, the time-keeper was subjected to pressures from 60 inches to 3 inches; and it will also be perceived, that the greatest deviation of the detached rate from the original rate, amounted only to 0".8. The observations embraced a period of four months, and during which the temperature varied from 39° to 60°

Detached + 7.9	Detached + 7.2
28.6 in. + 9.7	34 in. + 5.3
Detached + 8.0	Detached + 7.0
21 in. + 14.2	38 in. + 2.3
Detached + 7.9	Detached + 8.0
15 in. + 17.0	5 in. + 26.6
Detached + 8.2	Detached + 7.5
12 in. + 19.0	5 in. + 26.9
Detached + 7.8	Detached + 7.9
60 in. — 6.6	40 in. — 3.0
Detached + 8.7	Detached + 7.7
3 in. + 28.0	

The occasional imperfections of the valves of the condensing machine and of the air pump, have likewise afforded some interesting proofs of the truth of this investigation.

In more than one instance it has happened, that the mercurial column of the gauge of the air pump has been elevated during the intervals of comparison, by the introduction of air; and the effect of which was in all cases, an alteration of rate in the time-keeper, dependent on the degree of change. For example, during the period when a pocket chronometer was subjected to air of a density represented by 12 inches of quicksilver, and when its rate was  $+ 10''.0$ , the gauge became imperfect, the quicksilver rising to 18 inches soon after the time of comparison; the succeeding day the rate was  $+ 7''.5$ . Another chronometer under the same circumstances also underwent an alteration in its rate from  $+ 18''.0$  to  $+ 14''.2$ . In another instance, the rate of the time keeper was  $+ 15''.0$  under a pressure of 4 inches; during the next twenty-four hours the quicksilver rose to 12 inches, and the rate declined to  $+ 12''.4$ . The next day the imperfection of the gauge still continuing, the mercury rose to  $17\frac{1}{2}$  inches, and the rate sunk to  $+ 9''.9$ .

Sometimes a part of the air contained in the condensing engine, by escaping, would occasion fluctuations in the rates of the time keepers; but on restoring the machine to perfect order, they would immediately assume an uniform character. Thus, when a box chronometer was placed in air of uniform density, corresponding to 45 inches of mercury, the rates for three days were respectively  $- 10''.9$ ,  $- 10''.6$ , and  $- 10''.5$ ; but on attempting to increase the density to 48 inches of quicksilver, the machine disclosed proofs of imperfection; and the rates for three days were respectively  $- 15''.0$ ,  $- 12''.8$ , and  $- 10''.1$ . The machine was then repaired, and air introduced into it of a density denoted by 51 inches

of the mercurial column, when the daily rates were — 16".0, — 15".7, and — 16".0. In another instance, a time-keeper maintained a steady rate of — 10".5 under a pressure of 40 inches. During one day some of the air escaped, and the rate became — 7".0. The machine was then repaired and the former pressure restored, when the rate returned to — 10".3. On the third day air again escaped, and the rate changed to — 8".5; but on the fourth day the condenser being in perfect order, the time-keeper again returned to — 10".7. Thus, even from the temporary imperfections of the air pump and condensing machine, may proofs be drawn, demonstrating the effects of atmospheric pressure on the rates of chronometers.

An experiment was also attempted, at the suggestion of Mr. DAVIES GILBERT, to discover if, by removal of the case of a chronometer, and presenting its balance and spring to the free action of the air, the effect on the rate would present a result analogous to that which it would furnish under ordinary circumstances. With the case of the time-keeper in its proper situation, and the full pressure of the atmosphere on the machine, the rate of six days was — 15".3; but under the receiver with the mercurial gauge at 26 inches, it became for a like period — 13".6. By removing the case, the detached rate became — 16".4; and under the receiver, with the pressure of 26 inches, the rate was — 15".8; the removal of the case having in both instances increased the losing rate of the machine. On placing the time-keeper in the condensing machine under a pressure of an atmosphere and a quarter, the case being still removed, the rate was found to be — 15".5, differing but little from that determined in its

detached state. By augmenting the quantity of air to an atmosphere and a half, the daily variation still presented the same quantity ; but on removing the chronometer into the receiver under a pressure of 13 inches, the rate became  $-13''.3$  ; and with a pressure of 12 inches,  $-8''.8$ . Hence it appears, though the experiment is too limited to draw from it any general conclusion, that in this particular case, no alteration of rate took place by increasing the density of the air above its average state ; but by diminishing it *below* the same point, the rate was accelerated.

It may be proper however to advert to a circumstance, which may be regarded as having possibly exercised some influence on the foregoing results ; and that is, the change of temperature always attendant on sudden alteration in the density of the air.

When the chronometers were placed, for example, beneath the receiver of the air pump, a sudden diminution of temperature was in all cases produced, proportional to the degree of exhaustion ; and on restoring the medium to its original density, an elevation of temperature resulted, equally rapid in its effects. So also when the time-keepers were placed in the condensing engine, the sudden compression of the atmosphere, produced an immediate liberation of caloric, and which was followed, when the equilibrium of air was renewed, by a depression of temperature below its original condition.

These alterations of temperature, although producing a change of only two or three degrees, in a delicate thermometer, are nevertheless regarded by Mr. DALTON as the effect of a much greater degree of heat ; but which is permitted to impart its influence only for a very few seconds, in

consequence of the immediate effort made by the receiver, and the surrounding objects, to restore the primitive temperature. Now, although a sudden change of temperature, of forty or fifty degrees, if allowed to maintain its influence for any considerable time, would perhaps in some cases produce a small derangement of rate ; yet, from its continuing to act only for a few seconds, and producing, even on a sensible thermometer only a very small effect, it cannot be supposed that the adjustments for temperature in the balance of a chronometer, protected as they are by the thick case of the machine, can be in the least degree influenced by it. Accordingly, on introducing a very susceptible time-keeper, H, into an atmosphere  $50^{\circ}$  warmer than the ordinary state of the air, for ten seconds, no visible alteration of rate resulted ; and it may hence may be inferred, that the changes of temperature which have taken place during the preceding experiments, in consequence of variations in the density of the air, can have had no share in producing those changes of rate which have been perceived ; and that they resulted from alterations of pressure alone.

A change of rate in a chronometer, from an alteration in the density of the medium in which it is placed, considered as a simple fact, seems demonstrated from the foregoing experiments. Different hypotheses may probably be advanced respecting the cause ; but, the supposition which appears the most probable is, that a change takes place in the arc of vibration of the balance, in consequence of the altered density of the air, and a consequent variation in the rate of the timekeeper, from the imperfect isochronism of the balance.

The true measure of time is derived from the balance, and its vibrations will be isochronous, when the adjustments of the spiral spring are such as to admit of its elastic force varying directly with the arcs of vibration. As a necessary consequence of this principle, the application of any disturbing force will occasion no derangement in the rate, since the arcs of vibration, whether increased or diminished, must all be performed in the same absolute time.

It may however be questioned, whether a chronometer ever existed, in which the elastic force of the spring varied precisely with the arcs of vibration ; for it has been shown by Mr. ATWOOD,\* that though the weights employed to counterpoise the elastic force of a spring, at different angular distances from its quiescent position, may *appear* to be in the *exact* ratio of those distances, discrepancies too small to be detected by the nicest experimenter may exist, but yet be considerable enough, from the delicate nature of the machine, to create a sensible alteration of rate.

Any change in the arc of vibration of the balance of a chronometer thus constituted, must therefore be attended with some alteration of rate ; and that it is extremely probable such a change must take place when a time-keeper is placed in air of different densities, the arcs *increasing* when the density is *diminished*, and *diminishing* when it is *increased*, may be inferred, from the extreme delicacy of the balance, and its spring, and their consequent susceptibility of change.

The daily aberration of a chronometer, arising from a

\* Philosophical Transactions for 1794.

change in the arc of vibration, has been demonstrated by Mr. ATWOOD to be the following function.

$$24^h \left\{ \left( \frac{a}{a'} \right)^{\frac{-n}{2}} - 1 \right\}$$

in which  $a$  denotes the primitive arc of vibration,  $a'$  the arc resulting from the action of a disturbing force, and which, in addition to the case assumed by the abovementioned philosopher, may be regarded as either greater than  $a$ , or less than it; and  $n$ , the exponent, denoting the ratio between the elastic force of the spring, and the angular distances from the point of quiescence.

Supposing the primitive arc constant, the above function will undergo different modifications, according to the values assigned to the elements  $a'$  and  $n$ .

In the first place, we may conceive the elastic force of the spring to vary directly with the angular distances from the point of quiescence, the exponent  $n$  being, in this case, denoted by unity. This supposition, by causing the whole function to vanish, will indicate a perfect isochronism; and that therefore, whatever magnitude be attributed to the arc  $a'$ , in consequence of the action either of rarefied or condensed air, no alteration of rate will result. And it is remarkable that, during the whole course of experiments, I have not met with an instance to illustrate this case. Every chronometer examined exhibited a tendency either to gain or lose, when the density of the air was changed; and which therefore proves, that perfect isochronism is seldom if ever attained in the construction of a time-keeper.

Secondly, we may assign to the exponent  $n$ , a value *less*

than unity ; or, which is the same thing, we may suppose the elastic force of the spring to vary in a *less* ratio than that of the angular distances from the point of quiescence ; and which hypothesis will produce different results, according to the values assigned to  $a'$  ; for if the time-keeper be placed in condensed air, so as to make  $a'$  less than  $a$ , a *positive* value will be given to the function, or the chronometer will *gain*. If, on the contrary, the time-keeper be placed in rarefied air, so as to make  $a'$  greater than  $a$ , still preserving the magnitude of  $n$ , the numerical value of the formula will be *negative*, and the chronometer *lose*. Cases to illustrate both suppositions are recorded among the experiments.

Thirdly, we may suppose the elastic force of the spring to vary in a greater ratio than the angular distances from the point of quiescence, and in which case  $n$  must be greater than unity. If then we suppose the chronometer to be placed in condensed air, the value of  $a'$  becoming in such a case *less* than  $a$ , the numerical value of the function will be *negative*, and a retardation of rate will be the necessary result. But if the time-keeper be placed in rarefied air, so as to make  $a'$  *greater* than  $a$ , preserving the same value of  $n$ , the numerical result of the formula will be *positive*, and the time-keeper must *gain*. And this, according to the foregoing experiments, is by far the most general condition of chronometers. It may also be inferred from the same circumstance, that time-keepers are more frequently constructed with the elastic forces of their springs, varying in a *greater* ratio than the angular distances from the point of quiescence, than the contrary.

The preceding suppositions will therefore explain why



some chronometers, during the preceding experiments, *gained* with *diminished* pressure, and *lost* with *increased*; whilst others possessed properties precisely the reverse.

In determining the rates of the time-keepers, I had the advantage of a transit instrument, and an astronomical clock.

*Plymouth, March 1, 1824.*